

Four-manifolds with right-angled Artin fundamental groups

Alyson Hildum

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It is well known in low-dimensional topology that given a finitely presented group G , one can always find a 4-manifold M with fundamental group G . (There is a standard construction, which I will describe.) One of the items on Kirby's problem list is to find the minimum Euler characteristic of 4-manifolds with prescribed fundamental group G . This is a subproblem of the more general "geography problem" for a group G , in which one hopes to determine all possible values of the signature and Euler characteristic of M with fundamental group G .

In this talk I will focus on 4-manifolds with fundamental groups belonging to a certain class of groups called right-angled Artin groups (or RAAGs, as they are often called). RAAGs are a popular study among geometric group theorists today as they have rich subgroup properties. But they are also very easily presented. RAAGs are also known as graph groups because their presentations can uniquely be defined by graphs, where each vertex represents a generator and each edge between vertices represents a commutator relation between the associated generators. It is not difficult to construct a 4-manifold which has a particular RAAG as its fundamental group, however for most RAAGs, the "standard" construction is not minimal (i.e. the Euler characteristic is not minimal). I will give upper and lower bounds on the minimal Euler characteristic, and will then focus on tools for constructing minimal 4-manifolds. (This was the subject of my PhD thesis.)