Mathematical Crystals and Quasicrystals: Solid-to-Solid Phase Transitions

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In the early 1980's Dan Schectman made the Nobel Prize wining discovery of quasicrystals. These objects posses strikingly similar properties, especially long range order, to physical crystals (which are defined by a periodic molecular structure) but have a distinctive non-periodic molecular structure. An almost universal mathematical model for quasicrystals are the so called cut-and-project sets. The vertices of the Penrose tiling, for instance, is an example of such a set. It is a fundamental question to determine whether a given quasicrystal can be obtained by a displacive, as opposed to a diffusive, phase transition from a crystal. That is, can a quasicrystal be obtained by taking a crystal and applying a perturbation to it which moves each atom a uniformly bounded distance? We will show that in most moduli spaces of cut-and-project quasicrystals that (1) a quasicrystal can almost surely obtained from a crystal via a displacive phase transition, and (2) there is always a topologically large (i.e. residual) subset of quasicrystals that cannot be realized in this way. The results are obtained by relating cut-and-project sets as return times to a section for linear toral flows and employing cutting edge techniques from Fourier analysis, dynamics, and Diophantine approximation. This is joint work with Alan Haynes and Barak Weiss.