

Elliptic curves, torsion subgroups, and uniform bounds for Brauer groups of K3 surfaces

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Elliptic curves are smooth plane curves defined by a homogeneous equation of degree three that come with a marked point. Results on elliptic integrals going back to Euler show that one can endow such a curve with an abelian group structure, making the marked point the origin of this group. Mordell showed in 1922 that if E is an elliptic curve defined by an equation over the rational numbers \mathbb{Q} , then the group of points $E(\mathbb{Q})$ is finitely generated. Surprisingly, there are only 15 possibilities for the torsion subgroup of $E(\mathbb{Q})$. This is a spectacular theorem of Mazur from 1977. I will explore this circle of ideas for a higher dimensional analogue of elliptic curves: K3 surfaces. Unlike “abelian surfaces”, K3 surfaces have no group structure, so even understanding what the analogue of $E(\mathbb{Q})$ should be is tricky. I will explain how the Brauer group of K3 surface comes to the rescue, argue for a conjecture along the lines of Mazur’s theorem, and explain the impact this would have in our understanding of K3 surfaces.