Determinants, polynomials, and matroids

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November 5, 2015

Writing a multivariate polynomial as the determinant of a matrix of linear forms is a classical problem in algebraic geometry and complexity theory. Requiring that this matrix is Hermitian and positive definite at some point puts topological and algebraic restrictions on the polynomials that appear as the determinant and its minors. In particular the real zero sets of these polynomials are hyperbolic (or real stable) and interlace. Such polynomials appear in many areas of mathematics, including optimization, combinatorics and differential equations. Recently, tight connections have been developed with combinatorial objects called matroids. I will give an introduction to some of these objects and the fascinating connections between them.